

Multi-Firm Entrepreneurship and Financial Frictions

Chanont Banternghansa*

University of Chicago

Job Market Paper

[Link to the latest version](#)

February 8, 2017

Abstract

An entrepreneur's ability to save is crucial to mitigating aggregate productivity losses caused by underdeveloped financial markets. Previous studies of this mechanism assume that an entrepreneur's savings come from income generated by only one firm. In contrast, this paper uses a large, novel dataset from Thailand and, using a legal mandate that Thai households have unique surnames, documents a large share of entrepreneurs with income from multiple firms. They can therefore accumulate wealth from various sources, allowing financially constrained firms that are owned by multi-firm entrepreneurs to grow faster and survive longer than those owned by single-firm entrepreneurs. Motivated by these facts, I develop a tractable model of multi-firm entrepreneurship in the presence of financial frictions and study its impact on aggregate productivity and the allocation of capital. After calibrating to match the salient features of the Thai data, I find that the aggregate productivity loss due to financial frictions would rise from 7% to 21% if entrepreneurs could not own multiple firms.

Keywords: financial frictions, entrepreneurship, aggregate productivity, growth

JEL Codes: E44, G32, L26, O4, O16

*Contact: chanont@uchicago.edu. I am indebted to my advisors Chang-Tai Hsieh (chair), Francisco Buera, and Robert E. Lucas, Jr., for their encouragement and support. I thank Nancy Stokey for her advice and guidance. For helpful comments and discussions, I thank Jon Adams, Ufuk Akcigit, David Argeno, Philip Barrett, Martin Beraja, Elisa Giannone, Kerstin Holzheu, Erik Hurst, Anil Kashyap, Munseob Lee, Claudia Macaluso, Sara Moreira, John Nash, Nuno Paixao, Fabrice Tourre, Robert Townsend, Robert Shimer, Harald Uhlig, and Thomas Winberry, and Chen Yeh. Finally, I thank Yi Sun for her support on this project. All errors are my own.

1 Introduction

Multi-firm entrepreneurship has been understudied because panel data on firms with ownership information has been scarce, especially in developing countries. In this paper, I introduce a novel dataset from Thailand that sheds light on the importance of multi-firm entrepreneurship and its consequence on aggregate economy. Because Thai surnames are by law unique, I identify all firms owned by each entrepreneur, which would be a challenging task in other countries, where many surnames are common¹. The dataset also contains broad, detailed information about each firm, including its characteristics, financial statements, and dates of operation². These unique features of the dataset allow me to explore novel aspects of multiple-firm ownership in a developing country.

I find that multi-firm entrepreneurship has a significant impact on the economy as a whole, especially in overcoming underdeveloped financial markets. Papers such as [Evans and Jovanovic \(1989\)](#), [Buera and Shin \(2010\)](#), and [Moll \(2014\)](#) posit that the aggregate economy can reach near-optimal levels despite limited access to capital if entrepreneurs, given time, accumulate enough wealth from their businesses. These papers, however, assume that each entrepreneur relies on a single firm to overcome these constraints. But what are the consequences for the aggregate economy if entrepreneurs own and invest in *multiple firms*? What are the benefits of owning multiple firms and how does it affect firm dynamics? These questions have been left unanswered due to the lack of entrepreneur-firm level data and a parsimonious theoretical framework. My paper addresses these questions.

In this paper, I make two contributions. First, I construct a unique dataset that allows me to document new facts related to entrepreneurship and its effect on firms' growth. Second, I develop a tractable model of multi-firm entrepreneurship in the presence of collateral constraints. I use this theory and the dataset to quantify the effect of financial frictions on an

¹In Europe, the ORBIS/AMADEUS database contains ownership information but because of commonality in surnames, the matches could have measurement error. However, there have been algorithms to match entrepreneur names. See [Belenzon et al. \(2014\)](#) and [Belenzon et al. \(2015\)](#)

²The US Longitudinal Business Database has a long panel data of output and labor for each firm, but does not have capital, financial or ownership information.

economy with multi-firm entrepreneurs and describe the channels through which multi-firm ownership helps entrepreneurs overcome these constraints.

Through the lens of a new theoretical model, I find that entrepreneurs can save their way out of financial constraints more easily when they own multiple firms. My findings show that multi-firm entrepreneurs are able to survive longer, save more, and self-finance greater investments in their businesses. As a result, broader economic inefficiency due to financial frictions is attenuated. This savings channel has implications outside the Thai context as well. If multi-firm entrepreneurship in other countries is as common as in Thailand, then the effect of financial constraints on the aggregate economy in those countries should be mitigated. However, if entry costs for new firms in these countries are high, underdeveloped markets could still stifle productivity growth. Consequently, policies that reduce the entry barriers faced by firms will also reduce the effect of financial frictions.

I begin by constructing a new firm-entrepreneur dataset from Thailand, and I use this data to document new empirical facts about multi-firm entrepreneurship. This unique dataset makes Thailand an ideal setting in which to study financial frictions and multi-firm entrepreneurship because very few data have information on financial and ownership information. More importantly, due to the uniqueness of Thai surnames I can accurately identify the set of firms that each entrepreneurial family owns. Thus unlike others, my analysis studies the effects of financial frictions at the entrepreneur-level instead of the firm-level.

I observe several new empirical facts related to multi-firm entrepreneurship. I find that between 1999 and 2015, 35% of entrepreneurs owned multiple firms, a substantial share. I also find that financially constrained firms owned by multi-firm entrepreneurs experienced higher capital growth rates and lower exit rates, after controlling for initial wealth, productivity and firm size. I interpret this finding as evidence that ownership of multiple firms allows entrepreneurs to accumulate wealth more quickly and over a longer period of time.

I then proceed to use these facts to develop a tractable model that includes entrepreneur saving decision, endogenous choice of multi-firm ownership, and financial frictions in the

form of collateral constraints to quantify the impact of multi-firm entrepreneurship on the impact of imperfections in the financial market. In my model, an entrepreneur endogenously chooses the number of firms he operates in each period. Each firm experiences decreasing returns to scale, a per-period fixed cost of operation and a one-time set-up cost for each additional firm. Thus, entrepreneurs face a trade-off between fixed costs and revenue. Each firm's productivity is determined by its idiosyncratic productivity and its entrepreneur's productivity. Moreover, entrepreneurs are heterogeneous in wealth and productivity.

The novel feature of the model is that entrepreneurs can own multiple units of production. Since each firm's production function has decreasing returns to scale, entrepreneurs have an incentive to allocate capital to several firms despite the additional costs. The model also explores the role of diversification. If firm-specific shocks are not perfectly correlated, then ownership of multiple firms will reduce the volatility of an entrepreneur's income because diversification provides a hedge against large shocks. As a result, scale and diversification incentivize entrepreneurs to exit less frequently and accumulate more wealth over a longer period, thereby reducing the negative impact of financial frictions on aggregate economy.

The model also offers a parsimonious way to handle the large state space. If each firm has its own idiosyncratic shock, the curse of dimensionality makes computation challenging. However, because entrepreneurs care about their total income and not each firm's individual income, I show that the geometric sum of the firms' idiosyncratic shocks is a sufficient statistic for the firms' idiosyncratic shocks.

Finally, I calibrate my model to match the salient features of the Thai data and quantify how much multi-firm entrepreneurship can mitigate the adverse impact of financial frictions on the aggregate economy. I find that when entrepreneurs are allowed to have multiple firms, aggregate productivity loss is only 7%. In addition, I evaluate several counterfactuals scenarios to understand the roles of scale, diversification and selection. For example, when I shut down entrepreneur's ability to expand—an assumption used in past work—the aggregate productivity loss due to financial frictions triples to 21%.

Contribution to the Literature

My paper contributes to a myriad papers on financial frictions, entrepreneurship and growth. See [Matsuyama \(2008\)](#) for recent survey and also [Buera et al. \(2015\)](#) for a macro perspective.

First, with the availability of micro data, there has been a growing literature that examines distortion at the firm level. [Hsieh and Klenow \(2009\)](#) and [Hsieh and Klenow \(2014\)](#) show the presence of large misallocation in both capital and labor in India, China and Mexico. [Midrigan and Xu \(2014\)](#) use firm-level data from Korea to examine the role of financial frictions on entry and dispersions in returns to capital. [Gopinath et al. \(2015\)](#) show that TFP has been decreasing in Europe due to financial frictions. [Li et al. \(2015\)](#) examine the relationship between firm's borrowing capacity and its productivity using data from Japan. I contribute to this literature with a new firm-entrepreneur level dataset from Thailand which allows me to study the importance of multi-firm ownership. I document new empirical facts on multi-firm entrepreneurship, financial frictions and its impact on firm dynamics.

This paper also contributes to the literature that quantifies the role of financial frictions on growth. The relationship between wealth and entrepreneurship was introduced by the work of [Evans and Jovanovic \(1989\)](#). [Jeong and Townsend \(2007\)](#) provide an quantitative study between financial frictions, wealth accumulation and TFP. Recent works include [Buera et al. \(2011\)](#), [Buera and Shin \(2010\)](#), [Caselli and Gennaioli \(2013\)](#), [Midrigan and Xu \(2014\)](#), [Gopinath et al. \(2015\)](#), [Kerr et al. \(2015\)](#), and [Bah and Fang \(2016\)](#). In these papers, the role of wealth accumulation is crucial in determining effect of financial frictions on development and a crucial assumption is that entrepreneur and firm are considered identical. I introduce a parsimonious model in the spirit of [Hopenhayn \(1992\)](#), explore a new channel for entrepreneurs to self-finance by allowing entrepreneurs to endogenously pick multiple firms, and quantify this channel through the lens of my model.

I then examine the role of diminishing returns to scale technology and the ability to overcome the decreasing returns. With the exception of [Moll \(2014\)](#), many papers of financial frictions model production function or revenue in the form of decreasing, as motivated by

Lucas Jr (1978). Akcigit et al. (2016a) examine the ability to overcome decreasing returns in managerial time and its effect on TFP. My paper complements to that by examining the role of multiple firm ownership as a way to overcome decreasing returns.

My paper also examines the role of idiosyncratic shocks and self-insurance. Angeletos and Calvet (2005), Buera and Shin (2010), Covas (2006) and Moll (2014) examine the effect of idiosyncratic shocks on the the steady state. In my model, since firms' idiosyncratic shocks are not perfectly correlated, entrepreneurs can smooth their income by having multiple firms. This diversification benefit allows multi-firm entrepreneurs survive longer, which lengthen their time to self-finance and overcome financial frictions.

My paper also stresses the importance of using entrepreneur-level moments in addition to firm-level moments. Past papers use information on firm's exit rate to proxy for an entrepreneur's ability to self-finance. Buera et al. (2011) calibrate the probability that an entrepreneur draws a new productivity to the US manufacturing exit rate. In one of their extensions, Midrigan and Xu (2014) calibrate the per-period fixed cost to the age distribution of Korean firms. If an entrepreneur only owns one firm, then that firm's exit rate *is* the entrepreneur's exit rate which is defined as when an entrepreneur no longer operates any firm. If an entrepreneur owns multiple firms, however, then the exit rate of each firm does not necessarily represent the *entrepreneur's* exit rate. When there are multi-firm entrepreneurs, using only firm's exit rate to calibrate could lead to an inaccurate estimation of the effect of financial frictions on aggregate TFP.

The remainder of the paper is organized as follows. In section 2, I describe the unique set of panel data from Thailand that I have constructed, and I document empirical facts about multi-firm and single-firm entrepreneurs and the firms they own. In section 3, I develop the analytically tractable model of multi-firm entrepreneurship, the equilibrium conditions the consequent efficient allocations. In Section 4, I take the model to the data and I calibrate a set of parameters and show the effects of the two mechanisms on observed moments as well as the mitigation of aggregate TFP loss. Section 5 concludes.

2 Data Description

The data is collected from the Department of Business Development in Thailand. By law, registered companies must submit their tax returns twice a year. The dataset is compiled and cleaned by the author. It contains a 13-digit identifier number, name, registration and exit information of all new registered firms in Thailand since 1999 and any surviving registered firms before 1999. Registered firms have two main types of firms: partnership and limited liability. For firms that are registered partnership, the names of the shareholders and their share amounts are provided. For firms that are public or private limited liability, the names of the board of directors are provided. The names reflect the time that the data was collected³. Out of 1,292,322 firms in the dataset, about 61.5% are private limited, 38.4% are partnership and 0.1% are public limited.

The dataset has financial information on balance sheets, income statements, and profits of firms from 2007 to 2015. The amount of starting capital the firm has at its inception is also provided. Information on what years the firm submits its financial statements is provided. Each firm is classified at the Thailand Standard Industrial Classification (TSIC) based on its primary produced goods or services. These can be classified into 18 major industries based on the 4-digit ISIC codes.

The dataset is an ideal setting to study multi-firm entrepreneurs and financial frictions for several reasons. First, Thai family names are unique⁴ which allows me to identify each family and how many firms they owned in the dataset. Second, because most firms are private and financial information is provided, measures of financial frictions and wealth can be constructed for private firms, and not just firms listed publicly. Finally, firms are not only from manufacturing sector but also from services and agriculture. The variety of sectors allow me to explore the degree of diversification based on each person's firms.

³The dataset was first collected in 2014. Therefore, shareholders and board of directors information for partnership and limited liability firms respectively reflect the year 2014. The data was collected again in 2015 and less than 2% of firms in the sample have changes in family ownership.

⁴Last names became a legal requirement in 1913. In 1962, the Person Name Act was passed which forbids the use of existing surnames.

The dataset contains information on firm’s registry date, which years it submitted its financial statements, and its operating status. The income statement contains data on total and main revenue, total cost of production, interest paid, administrative expense, tax, and profits. The balance sheet contains data on capital, current and non-current assets, current and non-current liability and equity from 2007 to 2015.

Although there is an indicator whether each firm is active or not, it does not specify the year that the firm cease to be active. I use the year in which the firms last report their financial information as the last year that they operate. For example if firm A’s last report was in 2005, then I classify that firm A becomes inactive in 2006. I infer the firm’s age by taking the difference between the accounting year and the date that the firm was registered.

Definition of an entrepreneur, firm ownership, and exit

I define an “entrepreneur” at the family level. Entrepreneurs, especially in developing countries, tend to operate in family firms and decision making is taken at the family level. For each entrepreneur, I find if their names appear as shareholders in partnership firms and/or as directors in private limited firms.

Complications can arise when firms contain multiple entrepreneurs with different surnames. It could be that these entrepreneurs are related but have different surnames, or that the entrepreneurs simply are not kins and jointly own the firm. I take a conservative approach and classify entrepreneur ownership of a firm if she owns more than 50% of the firm ⁵. For partnership firms, this is done by comparing the total shares hold by one family entrepreneur to other families. For private limited firms, since only information of the board of directors is given, I assume that the board of directors are also shareholders of the firm and assume equal weights in their ownership⁶. Using my metric, each firm is owned by at

⁵Bertrand et al. (2008) uses the highest percentage of ultimate ownership of a company which could be below 50%. Under their definition my results also hold and are available upon request.

⁶A large number of private limited firms have board of directors who all share the same last name. The analysis is robust when I limited my sample to only limited partnership firms and the results can be provided upon request.

most a family entrepreneur. An entrepreneur “exits” when she no longer has any active firms.

The table below shows the summary statistics of firms owned by single-firm and multi-firm entrepreneurs under the classification mentioned in the previous section. Firms that are owned by multi-firm entrepreneurs are larger in output and capital than those owned by single-firm entrepreneurs. In addition, they have higher wealth (equity) and also higher leverage. Firms owned by multi-firm entrepreneurs constitute about 45% in total number of firm-year observations.

Table 1: Firm Summary Statistics

Variables	Firms owned by Single-firm entrepreneur	Firms owned by Multi-firm entrepreneur	Other firms
ln(Value Added)	13.98 (1.53)	14.57 (1.70)	14.83 (2.06)
ln(Capital)	12.37 (2.48)	13.34 (2.82)	13.48 (3.06)
ln(Equity)	14.73 (1.27)	15.30 (1.55)	15.52 (1.91)
$\frac{\text{Capital}}{\text{Equity}}$	0.80 (4.87)	1.21 (6.31)	1.06 (5.72)
Age	10.04 (8.30)	12.62 (9.96)	11.52 (9.28)
Firm-Year Observation Share	297,358 0.22	620,916 0.45	459,526 0.33

Note: standard deviation in parentheses

Figure 1 shows the distribution of firm ownership by the entrepreneur. In 2015, about 65% of entrepreneurs have one firm, 17% have two firms, and about 16.5% have 3 or more. The total share of entrepreneurs with multiple firms in 2015 is around 35% and the number is consistent in other years as well. This is a significant amount of entrepreneurs in the dataset. To understand where these entrepreneurs expand to, table 2 shows the number of unique 1-digit ISIC firm for each type of entrepreneur. Among entrepreneurs that own two firms,

60% of them own firms that are in two different 1-digit ISIC sectors. When entrepreneurs have more than five firms, only 4.41% of the entrepreneurs have all firms in one digit ISIC sector. This is suggestive evidence that diversification may be a reason why entrepreneurs own a very diverse set of firms in the data.

Figure 1: Distribution of Firm Ownership

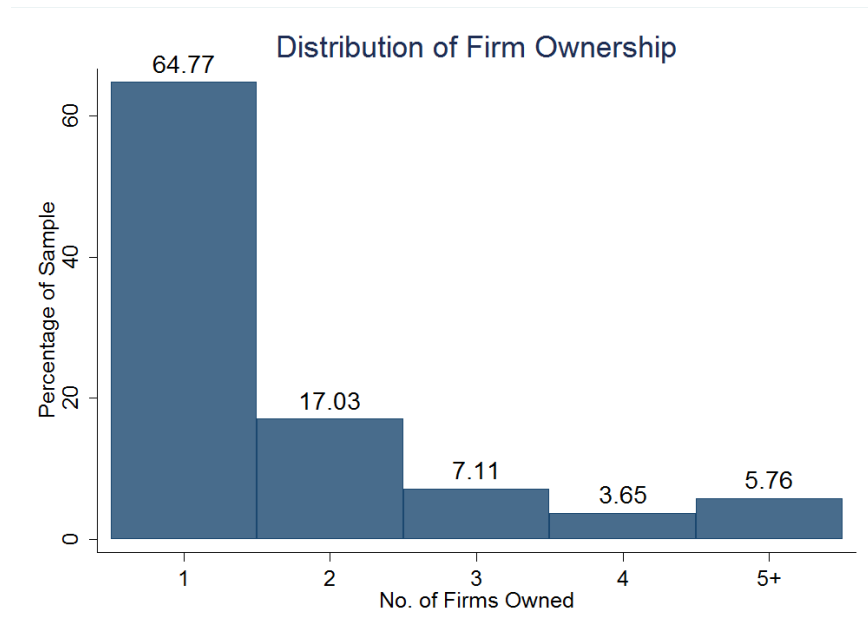


Table 2: Entrepreneurs and their type of firms

No of firms Owned	Obs.	Share of entrepreneurs with firms in j unique sectors				
		1	2	3	4	5+
1	113,926	100.00				
2	29,961	40.15	59.85			
3	12,512	20.39	52.2	27.41		
4	6,426	11.62	39.79	38.17	10.41	
5+	13,070	4.41	17.66	30.91	24.84	12.26

Note: Observations represent the number of entrepreneurs in the data, and they are binned by the number of firm each entrepreneur has. For each bin, I further separate them by the number of individual 1-digit ISIC firms that they own.

Exit rates of entrepreneurs and firms

In this section I look at the consequence of multi-firm ownership on the exit rates of entrepreneurs and their firms, as shown in figure 2. The blue solid line represents the firm exit rate while the red dash line represents the entrepreneur's. We see that entrepreneur's exit rates are lower than firm exit rates. Firm's exit rate is about 5% while entrepreneur's exit rate is about 2.28%. This is an important distinction because current literature uses firm's exit rate to proxy entrepreneur's exit rate. However, because entrepreneurs have multiple firms, their exit rates are in actually lower. This underlines the importance of using entrepreneur-level data in my calibration.

Figure 2: Firm and Entrepreneur Exit rates

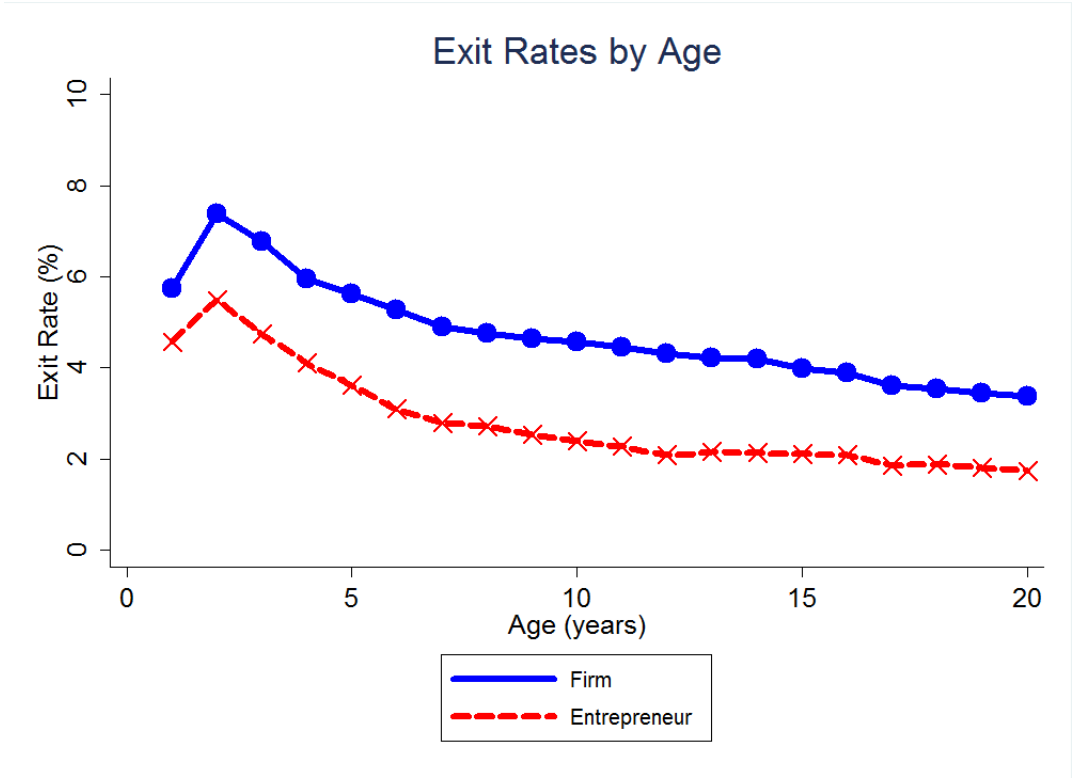
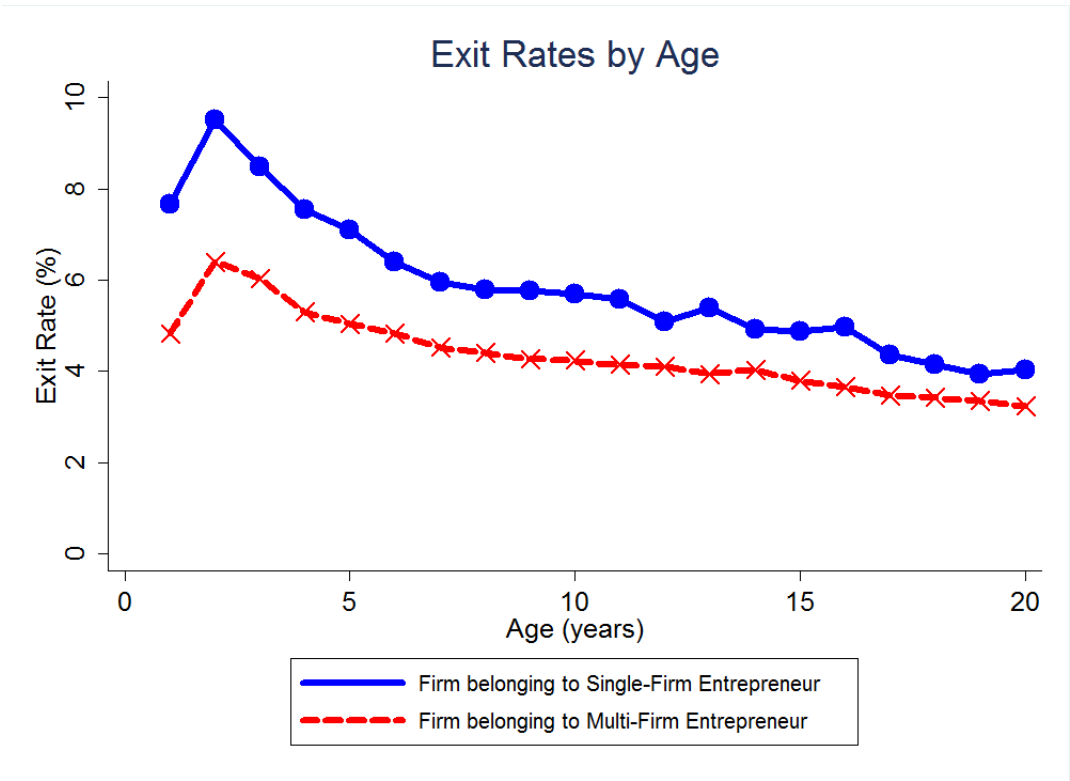


Figure 3 shows the exit rate of firm, differentiate by firm ownership of the entrepreneur. The blue solid line shows the exit rate of firms that belong to a single-firm entrepreneurs while the red dash line shows the exit rate of those that belong to multi-firm entrepreneurs. Interestingly, the exit rates of firms owned by multi-firm entrepreneur (4.83%) are lower than those owned by single-firm (5.72%). The difference in exit rates are robust after controlling for firm and entrepreneur characteristics (see figure 5 in appendix).

Figure 3: Firm Rates by Entrepreneur Ownership



Entrepreneur’s diversification and exit rate

To explain why firms owned by multi-firm entrepreneurs have lower exit rates than those owned by single-firm entrepreneur, I investigate if there’s a link between entrepreneur’s diversification and the firm’s exit rates. To measure entrepreneur’s portfolio, suppose an

entrepreneur has N firms with wealth W , then his portfolio return R_p is

$$R_p = \sum_{i=1}^N s_i R_i = \sum_{i=1}^N \frac{W_i}{W} \frac{\text{Profit}_i}{\text{Equity}_i}$$

where R_i is the return on equity of firm i and s_i is the share of entrepreneur's wealth invested in firm i such that $s_i = \frac{W_i}{W}$. The portfolio return variance is then $\sigma_p^2 = \text{var}(R_p) = \sum_i^N w_i^2 \sigma_i^2 + \sum_i^N \sum_{j \neq i}^N w_i w_j \text{cov}(R_i, R_j)$.

To see if entrepreneur's degree of diversification affect his firm's exit probability, I run a probit regression of probability that firm will exit on entrepreneur's measure of diversification. In the first specification, only entrepreneur's volatility is the independent variable. The result is reported in Table 3. Column 2 and Column 3 show that firm's profitability and firm's size has negative relationship with firm's exit rate, controlling for firm and entrepreneur characteristics. In column 4, I use the herfindahl index which is defined as $\sum_i^N s_i^2$ as an alternative measure of firm's diversification. In all columns, there is a positive relationship between portfolio volatility and firm's exit rate. This means that firm owned by an entrepreneur with a portfolio of high volatility is more likely to exit than a firm owned by entrepreneur with lower portfolio volatility. This is suggestive that diversification reduces the firm's exit rates and allow entrepreneurs to have smoother income.

Link between financial frictions and multi-firm entrepreneurship

This section shows a link between financial frictions and capital growth at the firm level. I will also show that the number of firms owned by an entrepreneur also increases capital growth at the firm level. Following [Gopinath et al. \(2015\)](#), I run the following regression:

$$\frac{k_{i,2014} - k_{i,2007}}{k_{i,2007}} = \alpha_0 + \alpha \log(a_{i,2007}) + \gamma \log(k_{i,2007}) + \beta \log(z_{i,2007}) + u_t$$

Table 3: Probability of firm exiting and entrepreneur's income volatility

	(1)	(2)	(3)	(4)
	P(Firm Exit=1)	P(Firm Exit=1)	P(Firm Exit=1)	P(Firm Exit=1)
Entrepreneur's Portfolio Volatility	0.457*** (5.00)	0.313*** (3.28)	0.295*** (3.21)	
Entrepreneur's Herfindahl				-0.181*** (-2.85)
Firm RoE		-0.106* (-1.92)	-0.114** (-2.17)	-0.171*** (-3.29)
Firm Asset		-0.0872*** (-6.96)	-0.108*** (-7.99)	-0.114*** (-8.25)
Other Controls				
Firm Age & Cohort	N	Y	Y	Y
Firm Industry	N	Y	Y	Y
Entrepreneur Exp& Cohort	N	N	Y	Y
Entrepreneur First Industry	N	N	Y	Y
Time Year Observation	538848	536201	534834	534834
Adj. R2	0.005	0.101	0.157	0.158

t statistics in parentheses
* $p < 0.10$, ** $p < 0.05$, *** $p < .01$

where the left-hand side is firm's i capital growth from 2007 to 2014. $a_{i,2007}$ is the firm's initial wealth in 2007, $k_{i,2007}$ is capital level in 2007, and $z_{i,2007}$ is firm's productivity. In a frictionless world, $a_{i,2007}$ should not have any predictive power since firm should be able to borrow as much as they want regardless of their wealth. Controlling for firm's productivity is also crucial because some firms may choose to be small, and ignoring firm's productivity will lead to incorrect estimation of the effect of financial frictions.

Since my data does not have labor, I supplement my data by merging with the Manufacturing Survey of Thailand in 2007, which contains detailed information on output, capital and labor at the establishment level. While the survey is rich at the establishment level, not all establishments have identifiers that allows me to match to my data, so measurement error could be a problem. I calculate $z_{i,2007}$ as TFPQ, following [Hsieh and Klenow \(2009\)](#).

The result is shown in table 4. In column 1, a 1% increase in equity leads to a 0.204% increase in capital growth and the estimate is statistically significant at the 0.1% level. This result is consistent with the findings in [Gopinath et al. \(2015\)](#) who find that a one percent increase in net worth leads to 0.17% increase in capital growth. As expected, productivity

is statistically significant and has a positive impact on capital growth, which means productive firms invest more. Column 2 shows that the results are robust controlling for sector and location fixed effects. In column 3, I added the number of firms that the entrepreneur owns as an additional independent variable. If firm's growth only depends on its initial wealth and productivity, then the number of firms the entrepreneur owns should have no impact. Instead, there is a positive relationship between the number of firms and firm's capital growth rate, and is statistically significant at the 1%. This suggest that financial frictions are present in the data and it has an impact on firm's capital growth. Second, multi-firm ownership also has a role in firm's individual capital growth, suggesting that firm owns by multi-firm entrepreneur grows faster and can overcome financial friction more easily.

Table 4: Capital Growth, Initial Net Worth and Number of firms

	(1)	(2)	(3)
	$k_{2014} - k_{2007}$	$k_{2014} - k_{2007}$	$k_{2014} - k_{2007}$
	k_{2007}	k_{2007}	k_{2007}
Ln(Equity)	0.204*** (0.04)	0.189*** (0.04)	0.167*** (0.05)
Ln(Productivity)	0.181*** (0.04)	0.238*** (0.04)	0.277*** (0.05)
Total No. of Firms owned by Entrepreneur			0.021** (0.01)
Ln(Capital)	-0.359*** (0.04)	-0.393*** (0.04)	-0.423*** (0.04)
Firm-Year Obs.	5193	5193	3608
Sector FE	N	Y	Y
Geographical Location FE	N	Y	Y

standard errors in parentheses
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

3 Benchmark Model

Time is discrete. There is a continuum of entrepreneurs that are heterogeneous in the number of firms N_t , wealth a_t , and productivity z_t . Entrepreneur's productivity evolves according to a persistent first-order AR(1) process in logs:

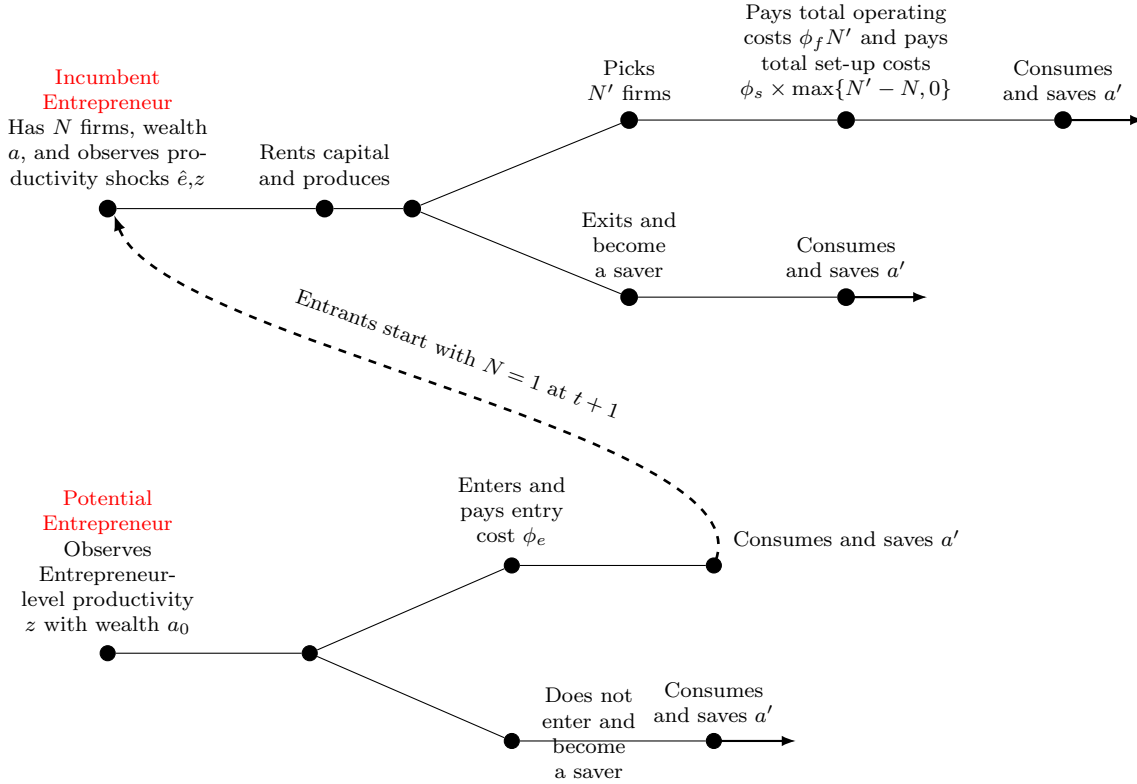
$$z_{t+1} = \mu_0 + \rho_z z_t + (1 - \rho_z^2) \sigma_z \epsilon_{z,t+1} \quad \text{with} \quad \epsilon_{z,t} \sim N(0, 1)$$

The conditional distribution of z_t will be denoted as $H(z_{t+1}|z_t)$. Among firms that are owned by the same entrepreneur, the firm-specific productivities $\vec{e} = \{e_1, e_2, \dots, e_M\}$ is driven by a multivariate normal process $\vec{e} \sim \mathcal{N}(\vec{0}, \Sigma)$ with distribution $F(\vec{e})$. For simplicity (although this is not necessary), I will assume that each firm has the same variance and the pairwise correlation is the same, such that $\text{var}(e_j) = \sigma_e^2$ for all j and $\text{corr}(e_j, e_i) = \rho$ for $i \neq j$. I set $\mu_0 = -\frac{\sigma_e^2 + \sigma_z^2}{2}$ so that $\mathbb{E}[\exp(z_t + e_1)] = 1$. Production of each firm is based on both the entrepreneur and firm-specific productivity, labor input, and their access to capital, which is limited by the entrepreneur's total wealth and the degree of financial constraint.

Entrepreneur can accumulate wealth a_t which has a risk-free return of r_t . In each period, an incumbent entrepreneur can choose to expand or reduce the number of firms that he want to operate next period, denote by N_{t+1} . The number of firms is chosen before productivity shocks z_{t+1} and \vec{e}_{t+1} are realized. Each firm incurs an operating fixed cost of ϕ_f and there is an one-time start-up cost for each additional firm ϕ_S . He can also choose to exit and he will become a saver.

Every period there is a constant mass $M > 0$ of prospective entrepreneurs, each start with an initial wealth a_0 and gets an entrepreneur-level productivity z drawn from a distribution $q(z)$. Conditional on entry, the distribution of the entrepreneur productivity in the first period of operation is $H(z'|z)$. Entering entrepreneurs start with one firm in the next period. If he does not enter he becomes a saver permanently. The timing is summarized in [Figure 4](#).

Figure 4: Timing in period t



3.1 Preferences

Entrepreneurs have CRRA preferences

$$U(c) = \mathbb{E} \left[\sum_{t=0}^{\infty} \beta^t u(c_t) \right] \quad (1)$$

where

$$u(c_t) = \frac{c_t^{1-\gamma}}{1-\gamma}$$

c_t is consumption at time t , β is the discount factor, γ is the coefficient of relative risk aversion (and the reciprocal of the elasticity of intertemporal substitution). The expectation is over the realization of the draw of entrepreneurial-level productivity z and firm-level productivities \vec{e} .

3.2 Technology

Each firm j that is owned by an entrepreneur faces a decreasing returns technology which produces output $Y_{j,t}$ using capital and labor as inputs:

$$Y_{j,t} = \exp(z_t + e_{j,t})^{1-\eta} (l_{j,t}^\alpha k_{j,t}^{1-\alpha})^\eta$$

where $\eta < 1$ is the span of control parameter, $\exp(z_t)$ is the entrepreneur-level productivity of the entrepreneur, $\exp(e_{j,t})$ is the firm-level productivity of firm j , and $k_{j,t}$ denotes the physical capital used by firm j at time t . $e_{j,t}$ is the j^{th} element in \vec{e}_t .

Firms rent capital at rate R where R is equal to the risk-free interest rate plus depreciation rate of capital. The profit of the entrepreneur with N firms is then:

$$\tilde{\pi} \equiv \sum_{j=1}^N Y_j - R \sum_{j=1}^N k_{j,t} - w \sum_{j=1}^N l_{j,t}$$

The borrowing constraint limits how much an entrepreneur can borrow as a function of his wealth. Entrepreneurs can borrow a total amount $K_t = \sum_{j=1}^N k_{j,t}$ less than or equal to a multiple of their wealth:

$$K_t \leq \lambda a_t \tag{2}$$

where $\lambda \geq 1$ governs the degree of financial frictions in the economy. The budget constraint of a entrepreneur is therefore

$$c_t + a_{t+1} = \tilde{\pi} - N_{t+1}\phi_f - \max\{N_{t+1} - N_t, 0\}\phi_s + (1+r)a$$

The entrepreneur uses his income as well as his wealth to finance his consumption, savings, fixed cost of his firms and any set up cost due to new firms.

3.3 Recursive Representation of the incumbent's problem

At the start of the period, an incumbent entrepreneur's state is summarized by his wealth a , productivity z , vector of firm-level productivity \vec{e} , and the number of firms N . The value function of an incumbent entrepreneur is represented as follows:

$$V(a, z, \vec{e}, N) = \max_{\substack{a', c, \\ N', \{k_j\}_{j=1}^N, \{l_j\}_{j=1}^N}} \begin{cases} u(c) + \beta \int_{\vec{e}'} \int_{z'} V(a', z', \vec{e}', N') dH(z'|z) dF(\vec{e}'), & \text{if } N' > 0 \\ u(c) + \beta V_s(a'), & \text{if } N' = 0 \end{cases}$$

$$\text{s.t. } c + a' = \tilde{\pi} - N' \phi_f - \max\{N' - N, 0\} \phi_s + (1 + r)a$$

$$\sum_{j=1}^N k_j \leq \lambda a$$

Based on a, z , and \vec{e} , he decides the allocation of $\{k_j\}_{j=1}^N$ and $\{l_j\}_{j=1}^N$ subject to his budget constraint and financial constraint. Based on the state variable in the current period, he decides how many firms he want to operate next period. When he decides he does not want to operate any firms ($N' = 0$), the incumbent entrepreneur “exits” and he becomes a saver. The value function of a saver is summarized by V_s :

$$V_s(a) = \max_{a', c} \{u(c) + \beta V_s(a')\}$$

$$\text{s.t. } c + a' = (1 + r)a$$

3.4 The entrant's optimization problem

The bellman equation of a prospective entrant with wealth a_0 and productivity z is

$$V_e(a_0, z) = \max \begin{cases} \max_{a', c} u(c) + \beta V_s(a') & , \text{saver} \\ \max_{a', c} u(c) + \beta \int_{\vec{e}'} \int_{z'} V(a', z', \vec{e}', 1) dH(z'|z) dF(\vec{e}') & , \text{enters} \end{cases}$$

$$\text{s.t. } c + a' = (1 + r)a_0$$

3.5 Characterization of the Incumbent Problem

Similarly to papers such as [Buera et al. \(2011\)](#) and [Midrigan and Xu \(2014\)](#), the entrepreneur observes the shocks before his deciding capital and labor, so the entrepreneur per period income is a static problem, which can be summarized by the following profit maximization problem:

$$\begin{aligned} \widetilde{\Pi}(a, z, \vec{e}, N) = & \max_{\{k_i\}_{i=1}^N, \{l_i\}_{i=1}^N} \left\{ \sum_{j=1}^N \exp(z_t + e_{j,t})^{1-\eta} (l_{j,t}^\alpha k_{j,t}^{1-\alpha})^\eta - R \sum_{j=1}^N k_{j,t} - w \sum_{j=1}^N l_{j,t} \right\} \quad (3) \\ \text{s.t. } & \sum_{i=1}^N k_i \leq \lambda a \end{aligned}$$

Taking the first order conditions of [equation \(3\)](#) with respect to l_i and k_i yield

$$\eta \frac{y_i(a, z, \vec{e}, N)}{l_i(a, z, \vec{e}, N)} = w \quad (4)$$

$$(1 - \alpha)\eta \frac{y_i(a, z, \vec{e}, N)}{k_i(a, z, \vec{e}, N)} = R + \theta(a, z, \vec{e}, N) \quad (5)$$

where $\theta_i(a, z, \vec{e}, N)$ is the multiplier of the borrowing constraint in [equation \(2\)](#). When the borrowing constraint is not binding, the marginal product of capital in [equation \(5\)](#) is equal to the rental rate of capital R . However when the borrowing constraint binds, the marginal product of capital is $R + \theta(a, z, \vec{e}, N)$, which is higher than the rental rate of capital. Thus, the presence of financial frictions create a wedge between the marginal product of capital and the rental rate of capital.

The challenge in [equation \(3\)](#) is that, since each firm has different firm-level specific shock, the state space \vec{e} increases with the number of firms an entrepreneur has. The next lemma will prove useful for the remainder of the analysis.

PROPOSITION 1 (Entrepreneur's optimal allocation): Entrepreneur's total capital demand,

total labor demand, total profit and the cutoff for being constrained depend on entrepreneur-level productivity z , his wealth a , and the geometric sum of firm-specific shocks \vec{e} :

$$\tilde{k}^E(a, z, \vec{e}, N) = \begin{cases} \left[\left(\frac{(1-\alpha)\eta}{R} \right)^{1-\alpha\eta} \left(\frac{\alpha\eta}{w} \right)^{\alpha\eta} \right]^{\frac{1}{1-\eta}} \exp(z) \sum_{j=1}^N \exp(e_j), & \text{if unconstrained} \\ \lambda a, & \text{if constrained} \end{cases} \quad (6)$$

$$\tilde{l}^E(a, z, \vec{e}, N) = \begin{cases} \left[\left(\frac{(1-\alpha)\eta}{R} \right)^{(1-\alpha)\eta} \left(\frac{\alpha\eta}{w} \right)^{1-(1-\alpha)\eta} \right]^{\frac{1}{1-\eta}} \exp(z) \sum_{j=1}^N \exp(e_j), & \text{if unconstrained} \\ \left[(\lambda a)^{(1-\alpha)\eta} \left(\frac{\alpha\eta}{w} \right) \right]^{\frac{1}{1-\eta}} \exp(z) \sum_{j=1}^N \exp(e_j), & \text{if constrained} \end{cases} \quad (7)$$

$$\tilde{\Pi}(a, z, \vec{e}, N) = \begin{cases} (1-\eta) \left[\left(\frac{(1-\alpha)\eta}{R} \right)^{(1-\alpha)\eta} \left(\frac{\alpha\eta}{w} \right)^{\alpha\eta} \right]^{\frac{1}{1-\eta}} \exp(z) \sum_{j=1}^N \exp(e_j), & \text{if unconstrained} \\ (1-\alpha\eta) \left[\left(\frac{\alpha\eta}{w} \right)^{\alpha\eta} (\lambda a)^{\eta(1-\alpha)} \left(\exp(z) \sum_{j=1}^N \exp(e_j) \right)^{1-\eta} \right]^{\frac{1}{1-\alpha\eta}} - R\lambda a, & \text{if constrained} \end{cases} \quad (8)$$

where the entrepreneur is constrained if and only if

$$\left[\left(\frac{(1-\alpha)\eta}{R} \right)^{1-\alpha\eta} \left(\frac{\alpha\eta}{w} \right)^{\alpha\eta} \right]^{\frac{1}{1-\eta}} \exp(z) \sum_{j=1}^N \exp(e_j) \geq \lambda a \quad (9)$$

(*Proof:* See Appendix 1). The condition in which firm j is constrained depends on $\sum_{i=1}^N \exp(e_i)$ and not individual e_j . This condition implies that if the entrepreneur own both firms and firm j is financially constrained, then firm $i \neq j$ will also be financially

constrained. Intuitively, since entrepreneur can freely reallocate capital among his firms, the marginal product of each firm must equalized.

In addition, the entrepreneur's total capital and labor demand do not depend on each firm's specific shock, but rather the geometric sum of firm-specific shocks $\sum_{j=1}^N \exp(e_j)$. Since the entrepreneur's decision is based on his *total* profit and not each firm's *individual* profit, $\exp(\hat{e})$ is a sufficient statistics to calculate entrepreneur's income. This is a great advantage because the state space for $\tilde{\Pi}(a, z, \vec{e}, N)$ is $N + 3$ but it can actually be summarized by a 4-variable state space, which I will denote by $\Pi(a, z, \sum_{j=1}^N \exp(e_j), N)$ as long as $\sum_{j=1}^N \exp(e_j)$ is known.

Unfortunately, $\sum_{j=1}^N \exp(e_j)$ —which is the sum of log normals—has no closed-form representation. However, sum of log normals can be reasonably approximated by another log-normal distribution [Fenton \(1960\)](#). We then have the following proposition:

PROPOSITION 2 (Sum of firm-specific shocks depends on *one* lognormal variable): The entrepreneur's total factor demands, profits, and the cutoff for being constrained given in [Proposition 1](#) depends on entrepreneur-level productivity z , his wealth a , and a log-normal variable \hat{e} :

$$\sum_{j=1}^N \exp(e_j) \approx \exp(\hat{e})$$

such that $\hat{e} \sim \mathcal{N}(\hat{\mu}(N, \rho), \hat{\sigma}^2(N, \rho))$ where

$$\begin{aligned} \hat{\sigma}^2(N, \rho) &= \ln \left[\frac{\exp(\sigma_e^2) + (N - 1) \exp(\rho \sigma_e^2)}{N} \right] \\ \hat{\mu}(N, \rho) &= \ln(N) + \frac{\sigma_e^2}{2} - \frac{\hat{\sigma}^2(N, \rho)}{2} \end{aligned} \tag{10}$$

with total profit:

$$\Pi(a, z, \hat{e}, N) = \begin{cases} (1 - \eta) \left[\left(\frac{(1 - \alpha)\eta}{R} \right)^{(1 - \alpha)\eta} \left(\frac{\alpha\eta}{w} \right)^{\alpha\eta} \right]^{\frac{1}{1 - \eta}} \exp(z + \hat{e}), & \text{if unconstrained} \\ (1 - \alpha\eta) \left[\left(\frac{\alpha\eta}{w} \right)^{\alpha\eta} (\lambda a)^{\eta(1 - \alpha)} \exp(z + \hat{e})^{1 - \eta} \right]^{\frac{1}{1 - \alpha\eta}} - R\lambda a, & \text{if constrained} \end{cases} \quad (11)$$

where the entrepreneur is constrained if and only if

$$\left[\left(\frac{(1 - \alpha)\eta}{R} \right)^{1 - \alpha\eta} \left(\frac{\alpha\eta}{w} \right)^{\alpha\eta} \right]^{\frac{1}{1 - \eta}} \exp(z + \hat{e}) \geq \lambda a \quad (12)$$

(*Proof:* See Appendix 1). With this tool, we can solve the incumbent problem

3.6 Stationary Competitive Equilibrium

Let $\mu_t^n(a, z, \hat{e})$ be the measure of entrepreneurs with firms n . Given input prices, a stationary competitive equilibrium is an invariant distribution of wealth and entrepreneurial productivity $G(A, z)$ and marginal distribution of z $dH(z'|z)$; policy function solves $c(a, z, \hat{e}, N)$, $a'(a, z, \hat{e}, N)$, $k(a, z, \hat{e}, N)$ and $N'(a, z, \hat{e}, N)$ such that:

1. $V_e(a_0, z)$ solves the entrant's problem.
2. $V_s(a)$ solves the saver's problem.
3. $\tilde{V}(a, z, \hat{e}, N, N')$ and $V(a, z, \hat{e}, N)$ solve the incumbent's problem.
4. Mass of entrepreneurs with N firm is fixed for all $N = 1, 2, 3$. $\mu_{t+1}^n(z', a', \hat{e}') = \int_{\hat{e}} \int_a \int_z P(z'|z) \mu_t(z, a, \hat{e})$
5. Number of entrants equals to the number of exiters.

3.7 Efficient Allocations

Let $i \in j$ index the entrepreneurs with firms j and let K and L denote the total amount of labor and capital used in the economy. Integrating the decision rules from [equation \(5\)](#) and [equation \(4\)](#) of entrepreneurs with firm j and then summing up all entrepreneurs yield the following expression for output:

$$Y = \frac{\left(\sum_{j=1}^N \left[\int_{i \in j} \exp(z_i + \hat{e}_i) (R + \theta_i)^{\frac{-(1-\alpha)\eta}{1-\eta}} di \right] \right)^{1-\alpha\eta}}{\left(\sum_{j=1}^N \left[\int_{i \in j} \exp(z_i + \hat{e}_i) (R + \theta_i)^{\frac{-1+\alpha\eta}{1-\eta}} di \right] \right)^{(1-\alpha)\eta}} (L^\alpha K^{1-\alpha})^\eta \quad (13)$$

where the first term of this expression yields the aggregate TFP given by the economy.

To calculate TFP in an efficient allocation, consider the social planner problem where he allocates capital and labor across these entrepreneurs,

$$\max_{k_i, l_i} \left(\sum_{j=1}^N \left[\int_{i \in j} \exp(z_i + \hat{e}_i)^{1-\eta} (l_i^\alpha k_i^{1-\alpha})^\eta di \right] \right)$$

subject to the constraint that the planner uses the same amount of aggregate labor and capital in the original allocation, and subject to the same number of firms each entrepreneur originally has. The social planner equalizes the marginal product of capital and labor across entrepreneurs, and the efficient output is

$$Y_e = \left(\sum_{j=1}^N \left[\int_{i \in j} \exp(z_i + \hat{e}_i) di \right] \right)^{1-\eta} (L^\alpha K^{1-\alpha})^\eta \quad (14)$$

Given that $R + \theta_i = \frac{y_i}{k_i}$, the TFP loss from the original economy is therefore given by

$$\text{TFP Losses} = \log \left(\sum_{j=1}^N \left[\int_{i \in j} \exp(z_i + \hat{e}_i) di \right] \right)^{1-\eta} - \log \frac{\left(\sum_{j=1}^N \left[\int_{i \in j} \exp(z_i + \hat{e}_i) \left(\frac{y_i}{k_i} \right)^{\frac{-(1-\alpha)\eta}{1-\eta}} di \right] \right)^{1-\alpha\eta}}{\left(\sum_{j=1}^N \left[\int_{i \in j} \exp(z_i + \hat{e}_i) \left(\frac{y_i}{k_i} \right)^{\frac{-1+\alpha\eta}{1-\eta}} di \right] \right)^{(1-\alpha)\eta}}$$

The expression for TFP loss is similar to [Midrigan and Xu \(2014\)](#). The two main difference is that entrepreneur's productivity is in the expression and that multiple-firms will increase TFP in the efficient misallocation. However, note that when entrepreneur expands from one to two firms, he now needs twice the amount of capital. Hence a single-firm unconstrained entrepreneur could become a multi-firm constrained entrepreneur when he expands, creating misallocation.

4 Parameterization

There are a total of 16 parameters. 7 parameters will be assigned exogenously based on the literature and the other 9 parameters are simultaneously set to match the salient features of Thai data.

4.1 External Parameters

Following [Buera et al. \(2011\)](#), I set the relative risk aversion parameter γ to 1.5 and the discount factor $\beta = 0.92$. The elasticity of capital in production function is set to be $(1 - \alpha) = \frac{1}{3}$. Capital depreciates at a rate $\delta = 0.06$. The span-of-control parameter η is 0.85 as in [Midrigan and Xu \(2014\)](#). Since the benchmark economy is a small open economy, the supply of capital to the entrepreneurs is inelastic. I set real interest rate to 4%, which is the average real interest rate of Thailand between 2000 to 2015. The wage rate is set to 1.

4.2 Calibrated Parameters

The remaining 9 parameters are jointly selected to match the moments of the Thai data. The entrepreneur-level productivity process follows an AR(1), which I discretize using the Rouwenhorst method. The firm-specific shock of a single-firm is independent and identically distributed but for firms owned by the same entrepreneur, the shocks are correlated at the cross section, which is governed by $\text{corr}(e_j, e_i) = \rho_e$. The firm idiosyncratic shocks are not correlated over time. I choose σ_z , the volatility of the entrepreneur-level productivity, ρ_z the persistence of the productivity, σ_e , the volatility of the firm-level shocks, and ρ_e , the correlation between the firm's shocks that are owned by the same entrepreneur, to match the standard deviation of log output in the data of 1.63, the autocorrelation of output of one period of 0.85, cross-sectional correlation of firms log output owned by the same entrepreneur of 0.68. Since some entrepreneurs own more than two firms, the correlation is calculated using the two largest firms owned by that entrepreneur.

Table 5 reports the persistence of the entrepreneur productivity. To match the high autocorrelation of log output in the data, ρ_z is equal to 0.982. The high persistence is needed because the firm-level idiosyncratic shock will lower the autocorrelation generated in the model. Given that the variance of entrepreneur productivity is 0.36 while the variance of firm-level idiosyncratic shock is 0.11, this implies that the volatility generated by the idiosyncratic accounts for 25% of the cross sectional variance of the productivity which is a significant portion of the firm's total level productivity.

The volatility implied by these parameters will also affect the exit rate of the entrepreneur. In addition, the firm's per-period fixed cost is $\phi_c = 0.12$, which pins down the average exit rate of entrepreneur of 2.28% per year and the share of young entrepreneurs of 0.21 in the data. The set up cost of additional firm is set to $\phi_s = 1.5$ to match the share of multi-firm entrepreneurs of 0.35 in the data.

To pin down the pareto tail parameter ν and initial wealth of entrant a_0 , I match the model to the relative size of entrant of 0.17 and exit rate of entrant entrepreneurs of 5.50%.

The relative size is calculated by taking the ratio of average output of entrant over the average output of the firm in the economy. Finally to match the average leverage in the data of 0.89, the strength of the rental market is set to $\lambda = 1.37$. Since the form of borrowing is through the rental rate of capital, which is done in one period, I abstract from using information on liabilities in the balance sheet. Instead I use the ratio $\frac{k_i}{a_i}$ in the data to match the moment generated by the model.

The model does well in matching the moments from the Thai data. The root mean square of the model is 0.038. The heart of the calibration is to get high persistence in the output of 0.85 over time but a cross-correlation of outputs to be 0.68, which is why the correlation in the firm-level idiosyncratic shocks are significantly negative.

Table 5: Parameter Values

Assigned Parameters	Symbol	Value
Discount factor	β	0.92
Relative Risk aversion	γ	1.5
Capital share	$1 - \alpha$	0.33
Span of control	η	0.85
Depreciation rate	δ	0.06
Interest Rate	r	0.04
Wage Rate	w	1
Calibrated Parameters		
Strength of rental market	λ	1.37
Persistence on entrepreneur-level productivity	ρ_z	0.982
Variance of entrepreneur-level productivity shock	σ_z^2	0.36
Variance of firm-level productivity shock	σ_e^2	0.11
Pair-wise correlation in firm-level productivity shock	ρ_e	-0.47
Firm Per-period fixed cost	ϕ_c	0.12
Additional Firm Set-up cost	ϕ_s	1.5
Scale Parameter of Pareto Distribution	ν	7.5
Initial wealth of entrant	a_0	0.66

Table 6: Moments

	Thai Data	Model
SD $\log y_t$	1.63	1.60
Corr($\log y_t, \log y_{t-1}$)	0.85	0.81
Corr($\log y_i, \log y_j$)	0.68	0.68
Relative Size of Entrant	0.17	0.16
Average Leverage $\frac{k_t}{a_t}$	0.89	0.97
Share of multi-firm entrepreneur	0.35	0.38
Share of entrepreneurs, age 1-10	0.21	0.25
Exit rate of entrant entrepreneur (in percent)	5.50	5.54
Exit rate of entrepreneur (in percent)	2.28	2.29

4.3 Aggregate Results

I compare the effect of financial frictions under different economies. In the first row, I rerun the model, restricting the entrepreneur to only have one firm. This is achieved by setting the set-up cost to be very high. The TFP from that economy is 1.58. In the second row, I allow the economy of the first row to equalize marginal product of capital, holding the aggregate capital and labor fixed. The TFP from that economy is 1.72. In the third row, I run the economy that allows entrepreneurs to have multiple firms, but no diversification benefits which yields TFP of 1.80. Then, I run the economy using the calibrated parameters from the previous section which yields TFP of 1.86. Finally, I allow the entrepreneurs to equalize marginal product of capital, holding the aggregate capital and labor constant. Comparing to the TFP of the efficient economy (last row), the TFP loss due to financial frictions when entrepreneur cannot have multiple firm is 20.7%. In contrast, the TFP loss in the calibrated economy — represented in row 4— experience a TFP loss of only 6.1%. If I shut down the diversification benefit—by allowing firm-specific shocks to be perfectly correlated—then the TFP loss is 9.1%, about 3% more than the benchmark economy. This implies that the benefit due to scale only accounts for about 80% while benefits due to diversification accounts for about 20%. Our calibration illustrates that, by having multiple firm in the economy, entrepreneurs can save their way out of financial frictions more easily.

Economy	Implied parameters	TFP	Loss
Single-firm economy	$\phi_s = \infty$	1.57	20.7%
Single-firm economy, reallocate capital	$\phi_s = \infty$	1.71	14.6%
Multi-firm economy, no diversification	$\rho_e = 1$	1.80	9.1%
Multi-firm economy with diversification	See table 5	1.86	6.1%
Multi-firm economy with diversification, social planner	See table 5	1.98	

5 Discussion

Timing of capital allocation

The benefit of diversification can be increased if the decision of capital is made *before* the entrepreneur and firm shocks are realized. This is the timing similar to [Angeletos and Calvet \(2005\)](#), [Covas \(2006\)](#) and [Gopinath et al. \(2015\)](#). As a result, idiosyncratic shocks will play a bigger role, which incentivize entrepreneurs to have multiple firms. In that setting, risk-averse entrepreneurs will engage in precautionary savings and there may even be an overaccumulation of capital in the economy. Therefore, although idiosyncratic shocks will have a bigger effect in this environment, over-accumulation of capital may instead lead to lower misallocation in the economy. In the appendix, I sketch a model of multi-firm entrepreneurs where the shock to the productivities are realized after the allocation of capital is decided.

Lack of persistence in firm's shocks

In this model, the firm's shocks do not have any persistence and is identically drawn over time. As a result, the persistence of entrepreneur's productivity needs to be high in order for the model to generate high autocorrelation yet a fairly large variance at the cross-section. Allowing persistence in firm's shocks and recalibrating to the model to the data will lead to lower persistence in entrepreneur's persistence. However, this comes at the expense of computational problem, since the shock of each firm now needs to be tracked.

Curvature of the production function

The main benefit of having multiple firms is escaping the decreasing returns to scale at each production unit. If entrepreneurial time is also limited, similar to [Akcigit et al. \(2016a\)](#), then the benefit of having multiple firms could decrease. In an extension, I added entrepreneurial time input that has diminishing returns. After calibrating to the data, the gain in having

multiple firms is indeed lower. However, because the benefits in having multiple firms are now decreasing in the number of firms, these entrepreneurs are now more sensitive to shocks. As a result, firms owned by these entrepreneurs exit more than those owned by single-firm entrepreneur, which is at odds with the data.

6 Conclusion

In this paper, exploiting the fact the Thai surnames are unique by law, I document new empirical facts on multi-firm entrepreneurs and their firms. I find that about 35% of entrepreneurs in the dataset has multiple firms. Not only do multi-firm entrepreneurs exit less than single-firm entrepreneurs, but their firms also exit less than those owned by single-firm entrepreneurs.

I develop a quantitative mode which allow entrepreneurs to own multiple firms as a new channel for them to overcome financial frictions. I use a unique dataset from Thailand to calibration certain parameters. Entrepreneurs can avoid the effect of decreasing returns to scale in production by operating in multiple firms. Operating in multiple firms is also seen as a form of diversification since exit rate for multi-firm entrepreneurs and their firms are lower than their counterparts. The parameterized model shows that multi-firm entrepreneurship can accumulate more wealth which on aggregate reduce the effect of financial frictions on TFP.

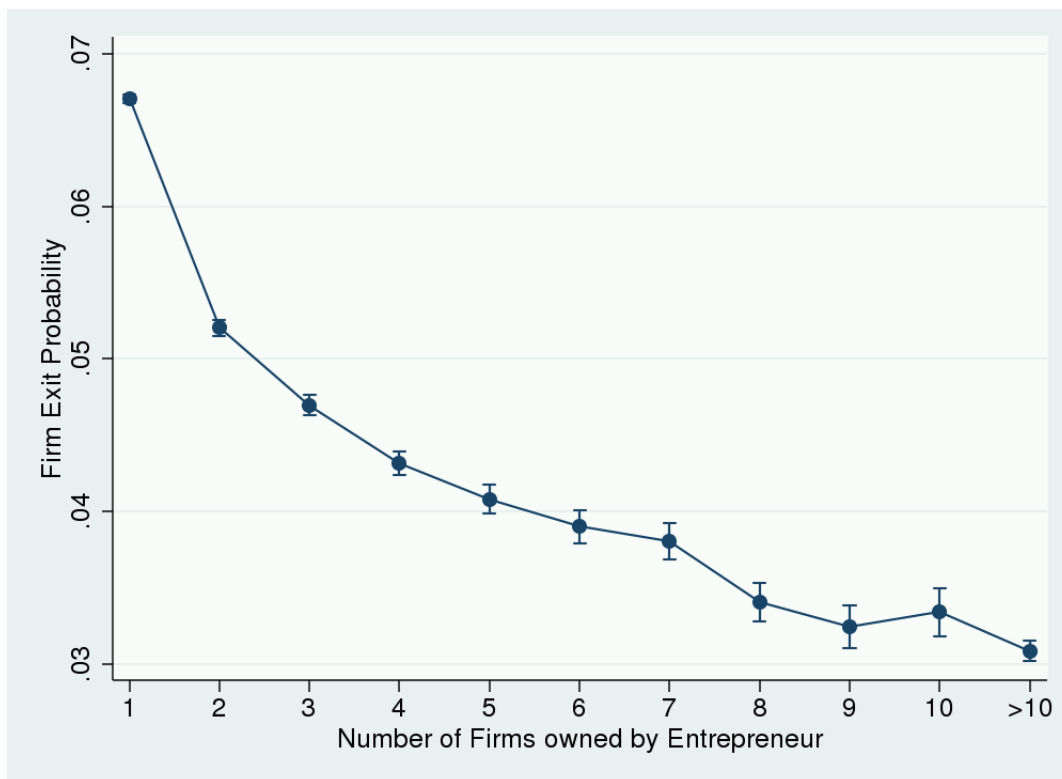
Given that multi-firm entrepreneurs are a large share of the entrepreneurs, and that they have an effect on firm dynamics, more consideration is needed to understand them. Future research is

A Appendix: Graphs and Tables

Table 7: Entrepreneurs who started in Wholesale (51) expanded into these sectors (in percent):

ISIC code	Sector	Percent
51	Wholesale trade, excl. Motor vehicles	27.64
52	Retail trade, excl. Motor vehicles	16.71
50	Sale of motor vehicles and motorcycles	3.93
55	Hotels and restaurants	2.28
70	Real estate activities	7.87
45	Construction	5.35
15	Manufacturing of Food/Beverages	2.55
63	Auxilliary transport activities	2.30
60	Land transport	2.23
29	Manufacture of Machinery and equipment	2.01
	Others	27.13

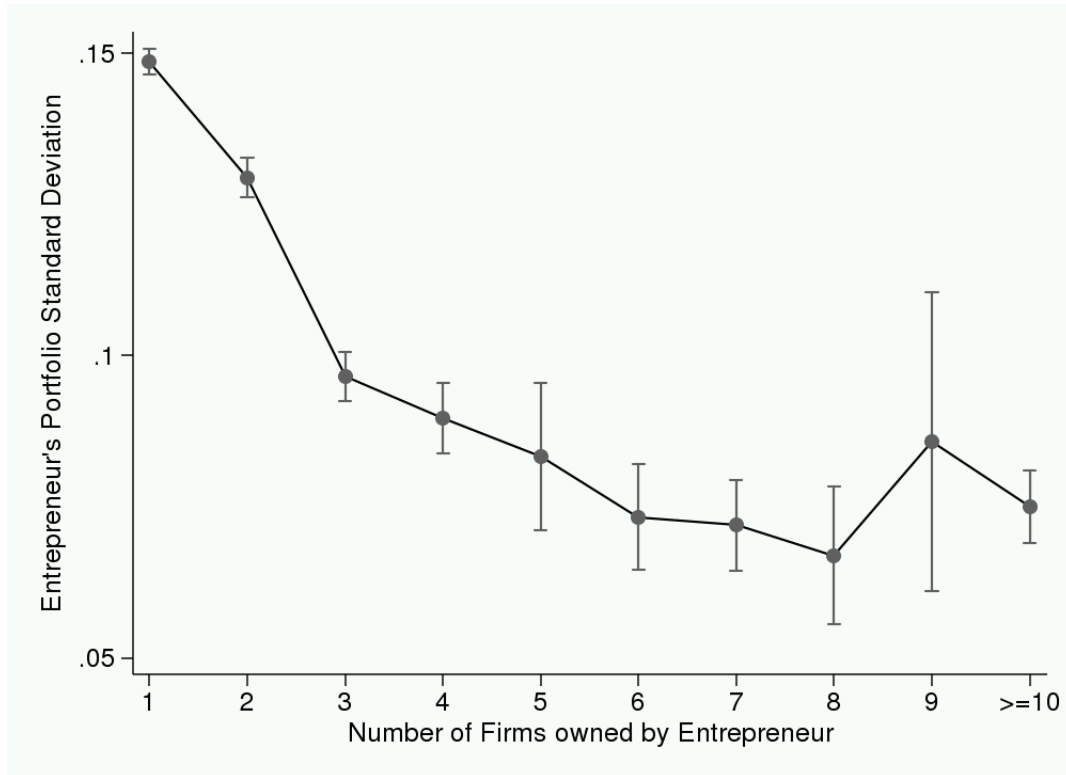
Figure 5: Annual Firm Exit Rate and Entrepreneur owned firms



I run a probit regression by regressing firm's exit rates on the number of firms an entrepreneur own, controlling for firm's age, firm's cohort, firm's 3-digit ISIC code and entrepreneur's experience. The firm exit rate is decreasing in the number of firms owned by entrepreneur.

The figure below plots the relationship between number of firms and entrepreneur's portfolio volatility, controlling for entrepreneur's characteristics⁷. The standard deviation on the portfolio return of a single-firm entrepreneur is about 15% which is quite high. The volatility decreases as entrepreneur owns more firms. When entrepreneur has eight firms, the standard deviation of her portfolio is 6.7%, less than half compare to those of single-firm entrepreneurs. By having multiple firms, entrepreneur can reduce the volatility on his portfolio return.

Figure 6: Entrepreneur Portfolio Return Volatility and Number of Firms



⁷The estimates are obtained by regression the entrepreneur's portfolio standard deviation on number of firms, weighted by total sales. I control for entrepreneur's experience, entrepreneur's cohort, and entrepreneur's starting sector. 95% confidence intervals are plotted.

Firm Life Cycle

To better understand how multi-firm entrepreneurship affects firm life-cycle over time, I consider an outcome variable X_{ijt} for firm i , entrepreneur j at time t . I run the following regression:

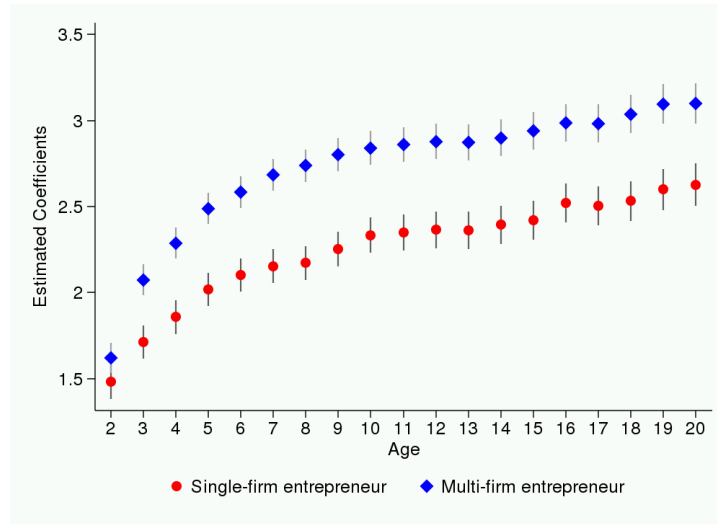
$$\ln(X_{ijt}) = \alpha_i + \beta_0 + \sum_i^{20} \beta_i \text{Age}_{it} + \sum_i^{20} \gamma_i \text{Age}_{it} \times \text{Multi-firm}_j + \epsilon_{ijt}$$

s.t. $\alpha_i = \alpha_C + \alpha_L + \alpha_G + \alpha_S$

where α_C are cohort-fixed effects, α_L are legal entity dummies, α_G are geographic dummies and α_S are firm 3-digit ISIC fixed effects.

Multi-firm $_j$ is a dummy variable to indicate if entrepreneur j owns multiple firm in the data. I am interested in the coefficients β_i and γ_i . β_i captures the age effect on outcome variable X_{ijt} for single-firm entrepreneurs, while γ_i captures the difference in age effect on outcome variable X_{ijt} between multi-firm and single-firm entrepreneurs. I consider the age effect on firm size and firm's leverage.

Figure 7: Life Cycle of Firm Size



B Appendix: Proofs of the Propositions

Proof of Proposition 1. Entrepreneur's Optimal Allocation

Maximizing [equation \(3\)](#) with respect to k_j yields

$$\tilde{k}_j(z, a, \vec{e}, N) = \begin{cases} \left[\frac{\nu}{R} \right]^{\frac{1}{1-\nu}} \exp(z + e_j), & \text{if unconstrained} \\ \lambda a \frac{\exp(e_j)}{\sum_{i=1}^N \exp(e_i)}, & \text{if constrained} \end{cases} \quad (15)$$

Note that financial constraint binds when $\left[\frac{\nu}{R} \right]^{\frac{1}{1-\nu}} \exp(z + e_j) > \lambda a \frac{\exp(e_j)}{\sum_{i=1}^N \exp(e_i)}$. This is when capital demand for firm j in the frictionless case is higher than what the entrepreneur can demand for firm j . Simplifying the inequality yields:

$$\left[\left(\frac{\nu}{R} \right)^\nu \right]^{\frac{1}{1-\nu}} \exp(z) \sum_{i=1}^N \exp(e_i) \geq \lambda a$$

Plugging [equation \(15\)](#) into firm's individual profit $\tilde{y}_j = \exp(z + e_j)^{1-\nu} (k_j)^\nu - Rk_j$ yields:

$$\tilde{y}_j(a, z, \vec{e}, N) = \begin{cases} (1 - \nu) \left[\left(\frac{\nu}{R} \right)^\nu \right]^{\frac{1}{1-\nu}} \exp(z + e_j), & \text{if unconstrained} \\ (\lambda a)^{1-\nu} \frac{\exp(z + e_j)}{\sum_{i=1}^N \exp(e_i)} - R\lambda a \frac{\exp(e_j)}{\sum_{i=1}^N \exp(e_i)}, & \text{if constrained} \end{cases} \quad (16)$$

Taking the sum of [equation \(15\)](#) and [equation \(16\)](#) from $j = 1 \dots N$ yields the solution to [equation \(6\)](#) and [equation \(8\)](#) respectively.

QED.

Proof of Proposition 2. Sum of firm-specific shocks depends on *one* lognormal variable.

Let L_i be a lognormal variable with location parameter μ_i and scale parameter σ_i and let the pair-wise correlation between L_i and L_j be ρ_{ij} for any $i \neq j$. The Fenton-Wilkinson approximation for $L = \sum_i^N L_i$ is obtained by matching the first two moments such that L follows a location parameter μ_z and σ_z^2 such that

$$\mu_z = 2 \ln(u_1) - \frac{1}{2} \ln(u_2)$$

$$\sigma_z^2 = \ln(u_2) - 2 \ln(u_1)$$

where

$$u_1 = \sum_i^N \exp\left(\mu_i + \frac{\sigma_i^2}{2}\right)$$

$$u_2 = \sum_i^N \exp(2\mu_i + \sigma_i^2) + 2 \sum_{i=1}^{N-1} \sum_{j=i+1}^N \left\{ \exp(\mu_i + \mu_j) \exp\left(\frac{1}{2} (\sigma_i^2 + \sigma_j^2 + 2\rho_{ij}\sigma_i\sigma_j)\right) \right\}$$

Setting $\sigma_i^2 = \sigma_j^2 = \sigma_e^2$, $\mu_i = \mu_j = 0$ and $\rho_{ij} = \rho$ yield the approximation of $\sum_{i=1}^N \exp(e_i)$ in [equation \(10\)](#).

Define $\exp(\hat{e}) \approx \sum_{j=1}^N \exp(e_j)$ and substitute $\exp(\hat{e})$ in [equation \(6\)](#), [equation \(8\)](#), and [equation \(9\)](#) to get [equation \(11\)](#) and [equation \(12\)](#).

QED.

C Entrepreneur's Expected Utility and Number of Firms in a static setting

Consider a static version of the benchmark model. Since firm-level idiosyncratic productivities \vec{e} can be summarized by one random variable \hat{e} , equation (11) also makes it simpler to interpret how the number of firms affect entrepreneur's total income and his expected utility. In order to build some intuition, I consider a frictionless world where entrepreneurs can borrow as much as they want, their entrepreneur-level productivity is $z = 0$, and each entrepreneur has zero wealth. Then the entrepreneur's utility depends only on his profit:

$$E_{\hat{e}}[U(c)] = E_{\hat{e}} \left[\frac{\left[\tilde{\Pi}(0, 0, \hat{e}, N) \right]^{1-\sigma}}{1-\sigma} \right]$$

Note that \hat{e} is a function of N and ρ , the pair-wise correlation of each firm. Define $\pi(N, \rho) \equiv \tilde{\Pi}(0, 0, \hat{e}, N)$. Then we have the following proposition:

PROPOSITION 3 (Entrepreneur prefers more and uncorrelated firms): Let $\pi(N, \rho)$ denote the income process with n firms and pair-wise correlation ρ . Then $\pi(N, \rho)$ follows a log-normal distribution with the following properties:

1. $\pi(N + 1, \rho)$ stochastically dominates $\pi(N, \rho)$ at the second order and

$$E[U(\pi(N + 1, \rho))] > E[U(\pi(N, \rho))]$$

for all $N \geq 1$, and $-1 \leq \rho \leq 1$.

2. For any $\rho_1 < \rho_2$, $\pi(N, \rho_2)$ is a mean-preserving spread of $\pi(N, \rho_1)$ and $\pi(N, \rho_1)$ stochas-

tically dominates $\pi(N, \rho_2)$ at the second order, such that

$$E[U(\pi(N, \rho_1))] > E[U(\pi(N, \rho_2))]$$

for all $N > 1$.

(*Proof:* See Proposition 3). In other words, entrepreneurs prefer more firms and prefer firms that are negatively correlated. This proposition provides a convenient way to rationalize why entrepreneurs are better off with more firms. By owning multiple firms, entrepreneurs expand his scale because the technology of each firm is decreasing returns to scale. For the same number of firms, risk-averse entrepreneurs always prefer firms that are less correlated. Intuitively, even though $\pi(N, \rho_1)$ and $\pi(N, \rho_2)$ yield the same mean, the entrepreneur's income is more diversified as firms are less correlated. This proposition gives us the parallel result in the classic portfolio theory, in which investor is better off as the number of uncorrelated securities increase in his portfolio.

In the dynamic setting in which there are fixed costs, financial frictions and wealth accumulations, entrepreneur's choice for multiple firms is not as clear cut as the static setting. In which case I will rely on numerical solution.

Proof of Proposition 3. Entrepreneur prefers more and uncorrelated firms.

Since $\pi(N, \rho)$ follows a log normal distribution. I need to show that $\pi(N+1, \rho)$ stochastically dominates $\pi(N, \rho)$ at the second order, which using Levy's theorem (1973), requires that

$$\begin{aligned}\hat{\sigma}^2(N+1, \rho) &\leq \hat{\sigma}^2(N, \rho) \\ \hat{\mu}(N+1, \rho) &\geq \hat{\mu}(N, \rho) \\ E[\pi(N+1, \rho)] &\geq E[\pi(N, \rho)]\end{aligned}\tag{17}$$

Using equation (10) yields

$$\begin{aligned}\hat{\sigma}^2(N+1, \rho) \leq \hat{\sigma}^2(N, \rho) &\iff \exp(\rho\sigma_e^2) \leq \exp(\sigma_e^2) \quad , \text{with equality if } \rho = 1 \\ \hat{\mu}(N+1, \rho) \geq \hat{\mu}(N, \rho) &\iff \ln(N+1) - \frac{\hat{\sigma}^2(N+1)}{2} > \ln(N) - \frac{\hat{\sigma}^2(N)}{2} \quad , \text{follows from above} \\ E[\pi(N+1, \rho)] \geq E[\pi(N, \rho)] &\iff \ln(N+1) + \frac{\sigma_e^2}{2} > \ln(N) + \frac{\sigma_e^2}{2} \quad , \text{for any } N \geq 2\end{aligned}\tag{18}$$

Therefore $\pi(N+1, \rho)$ stochastically dominates $\pi(N, \rho)$ at the second order.

Now we need to show that $\pi(N, \rho_1)$ stochastically dominates $\pi(N, \rho_2)$ at the second order for $\rho_1 < \rho_2$. Using equation (10) yields

$$\begin{aligned}\hat{\sigma}^2(N, \rho_1) \leq \hat{\sigma}^2(N, \rho_2) &\iff \exp(\rho_1\sigma_e^2) < \exp(\rho_2\sigma_e^2) \quad , \text{since } \rho_1 < \rho_2 \\ \hat{\mu}(N, \rho_1) \geq \hat{\mu}(N, \rho_2) &\iff \ln(N) - \frac{\hat{\sigma}^2(N, \rho_1)}{2} > \ln(N) - \frac{\hat{\sigma}^2(N, \rho_2)}{2} \quad , \text{follows from above} \\ E[\pi(N, \rho_1)] \geq E[\pi(N, \rho_2)] &\iff \ln(N) + \frac{\sigma_e^2}{2} = \ln(N) + \frac{\sigma_e^2}{2}\end{aligned}\tag{19}$$

As long as utility is increasing and concave (risk aversion), then the individual will select 1) more firms and 2) less correlated firms. **QED.**

D Appendix: Model where capital is owned instead of rented

This appendix presents a version of the model where entrepreneur owns capital. In addition, entrepreneurs decide the allocation of capital before the shocks are realized. This increases the incentives for entrepreneurs to diversify and adds another source of dispersion in capital.

Time is discrete. There is a continuum of entrepreneurs that are heterogeneous in productivity z_t and their wealth a_t . Individual's wealth is determined endogenously by forward-looking savings behavior. Entrepreneurs have CRRA preferences

$$U(c) = \mathbb{E} \left[\sum_{t=0}^{\infty} \beta^t u(c_t) \right] \quad (20)$$

where

$$u(c_t) = \frac{c_t^{1-\eta}}{1-\eta}$$

The expectation is over the realization of the draw of entrepreneurial-level productivity (z) and firm-level productivities (\bar{e}). Entrepreneur-level productivity is driven by an AR(1) process:

$$z_{t+1} = \rho_z z_t + \sigma_z \epsilon_{z,t}$$

where $\epsilon_{z,t} \sim N(0,1)$ for all $t \geq 0$. The conditional distribution of z_t will be denoted as $H(z_{t+1}|z_t)$.

Technology

Each firm j that is owned by an entrepreneur faces a decreasing returns technology which produces output $Y_{j,t}$ using capital as input:

$$Y_{j,t} = \exp(z_t + e_{j,t})(k_{j,t})^\alpha$$

where $\alpha < 1$ is the span of control parameter, $\exp(z_t)$ is the entrepreneur-level productivity of the entrepreneur, $\exp(e_{j,t})$ is the firm-level productivity of firm j , and $k_{j,t}$ denotes the physical capital used by firm j at time t . $e_{j,t}$ is the j^{th} element in \vec{e}_t . \vec{e}_t is driven by a multivariate normal process $\vec{e} \sim \mathcal{N}(\vec{0}, \Sigma)$ with distribution $F(\vec{e})$. Entrepreneur's total capital is $K_t = \sum_{j=1}^N k_{j,t}$. α governs the importance of scale while the correlation in Σ gives the benefit of diversification.

Borrowing Constraint

A firm can borrow up to a fraction of the firm's pledged capital. This ensures that each firm will be able to pay its debt in the next period. The amount of debt that the firm can borrow reflects the degree of financial frictions. The borrowing constraint is given by:

$$b_{j,t+1} \leq \frac{\lambda - 1}{\lambda} k_{j,t+1}$$

Entrepreneur's total debt is then $B_t = \sum_{j=1}^N b_{j,t}$. One cost of having multiple firm is that, if one firm has a higher productivity, entrepreneur cannot instantaneously reallocate capital from a lower productivity firm to a higher one. The net wealth of a firm is $a_{j,t} = k_{j,t} - b_{j,t}$.

Net Income and Timing

The entrepreneur earns his income from all his firms, denoted as $Y_t = \sum_{j=1}^N Y_{j,t}$. Each period he has to repay his debt at rate r . His income net capital depreciation and debt is as follows:

$$Y_t + (1 - \delta)K_t - (1 + r)B_t$$

Every period the entrepreneur decides how many firms he want to operate in the next period. He pays a fixed cost ϕ for each firm he wants to operate. In addition, the entrepreneur has to decide in this period how much capital $k_{j,t+1}$ he would like to allocate to each firm j in the next period. Then he decides how much debt each firm wants to borrow based on the

above borrowing constraint. Firm's investment is given by $i_{j,t} = k_{j,t+1} - (1 - \delta)k_{j,t}$. Finally, he consumes what is left of his income. If he decides that he no longer wants any firm, the entrepreneur quits and consumes all his income. The timing is summarized in [Figure 8](#).

Recursive Representation of the Entrepreneur's problem

$$V(z, \vec{e}, \{k_j\}_{j=1}^N, \{b_j\}_{j=1}^N, N) = \max_{\substack{c, N' \\ \{b'_j\}_{j=1}^N, \{k'_j\}_{j=1}^N}} \begin{cases} u(c) + \beta \int_{\vec{e}'} \int_{z'} V(z', \vec{e}', \{k'_j\}_{j=1}^N, \{b'_j\}_{j=1}^N, N') dH(z'|z) dF(\vec{e}'), & \text{if } N' > 0 \\ u(c), & \text{if } N' = 0 \end{cases}$$

subject to

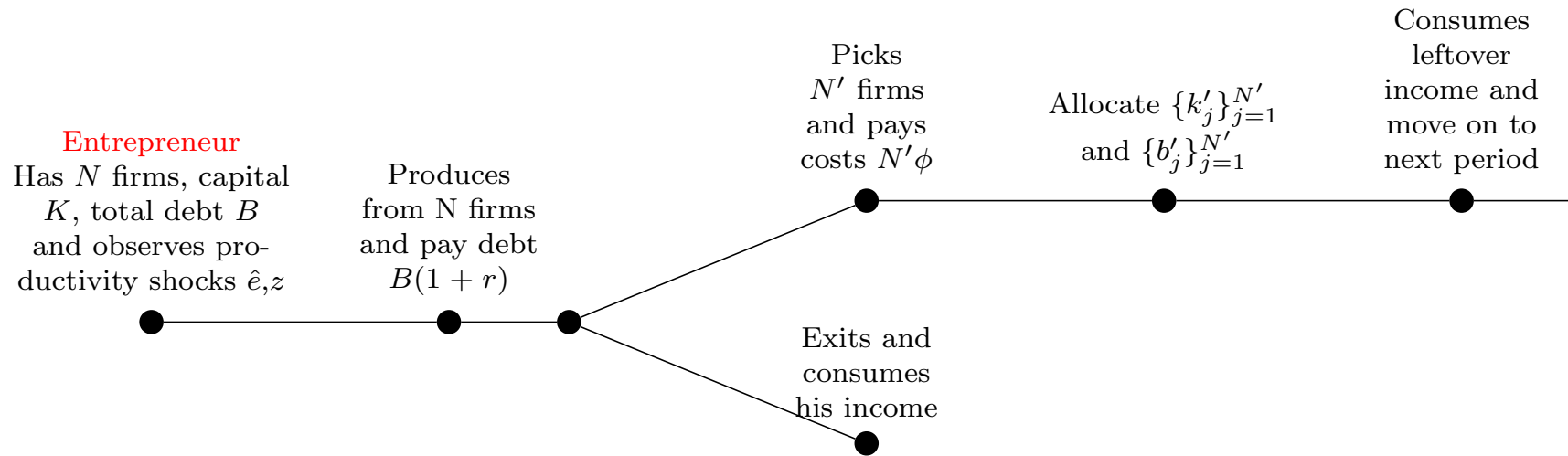
$$c + K' + B' = Y(z, \vec{e}, \{k_j\}_{j=1}^N, \{b_j\}_{j=1}^N, N) + K(1 - \delta) - (1 + r)B - N'\phi$$

$$b'_j \leq \frac{\lambda - 1}{\lambda} k'_j, \quad \text{for } j = 1, \dots, N'$$

$$B' = \sum_{j=1}^{N'} b'_j$$

$$K' = \sum_{j=1}^{N'} k'_j$$

Figure 8: Timing in period t



E Link between the entrepreneur's exit rate and aggregate TFP: A simple example

In this section, I illustrate how an entrepreneur's exit rate affects aggregate TFP in an environment with financial frictions. I use a simple framework similar to [Moll \(2014\)](#), which gives closed-form expression for the aggregate TFP. I will show under this setting that a higher exit rate directly reduces aggregate TFP.

E.1 Preferences and Technology

The economy is populated by entrepreneurs and hand-to-mouth workers. There is a mass of L workers who supply one efficient unit of labor inelastically. Entrepreneurs have heterogeneous talent z and wealth a . Each period the entrepreneurs draw a new productivity from a distribution $f(z)$. Entrepreneurs have preferences

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \log c_t$$

Each entrepreneur has access to the technology

$$y_t = (z_t k_t)^\alpha l_t^{1-\alpha}$$

Capital depreciates at the rate δ and is rented at rate $R_t = r_t + \delta$. The entrepreneur is subjected to a collateral constraint modeled as $k_t \leq \lambda a_t$. For each period, there is a probability p that the entrepreneur can produce. With probability $1 - p$, the entrepreneur will not be allowed to produce in that period. I interpret $(1 - p)$ as an entrepreneur's exogenous exit rate in that period and p as the survival rate of the entrepreneur.

E.2 Budgets

Entrepreneurs who are allowed to produce have wealth evolution according to

$$a_{t+1} = (z_t k_t)^\alpha l_t^{1-\alpha} - R_t k_t - w_t l_t + (1+r)a_t - c_t \quad (21)$$

Meanwhile, entrepreneurs who are hit by shock $(1-p)$ cannot produce this period, and their wealth evolves according to

$$a_{t+1} = (1+r)a_t - c_t \quad (22)$$

E.3 Entrepreneur's problem

In this setup, the entrepreneur's production decision is separate from their savings and consumption decision. The profit function is:

$$\Pi(a, z) = \max_{k, l} \{ (zk)^\alpha l^{1-\alpha} - Rk - wl \quad \text{s.t.} \quad k \leq \lambda a \}$$

The bellman equation V^P for the entrepreneurs who can produce is:

$$V^P(a, z) = \max_{c, a'} \log c + \beta \mathbb{E} [pV^P(a', z') + (1-p)V^I(a', z')] \quad (23)$$

subject to

$$a' = \Pi(a, z) + (1+r)a - c \quad (24)$$

V^I is the bellman equation for entrepreneurs who are exogenously hit by the shock $(1-p)$ and are inactive for that period

$$V^I(a, z) = \max_{c, a'} \log c + \beta \mathbb{E} [pV^P(a', z') + (1-p)V^I(a', z')] \quad (25)$$

$$\text{subject to } a' = (1 + r)a - c \quad (26)$$

The producing entrepreneurs' demands and profits are summarized as follows:

$$k(a, z) = \begin{cases} \lambda a & z \geq \underline{z} \\ 0 & z \leq \underline{z} \end{cases}$$

$$l(a, z) = \left(\frac{1 - \alpha}{w} \right)^{1/\alpha} z k(a, z)$$

$$\Pi(a, z) = \max \{z\pi - R, 0\} \lambda a, \quad \pi = \alpha \left(\frac{1 - \alpha}{w} \right)^{\frac{1 - \alpha}{\alpha}}$$

Note that, with constant returns to scale production function, profits are linear in wealth and there is a productivity cutoff \underline{z} in which active entrepreneurs with $z \geq \underline{z}$ produce. The cutoff is defined by $\underline{z}\pi = R$. The producing entrepreneur's net income is $[\max \{z\pi - R, 0\} \lambda + 1 + r] a$. However, the entrepreneurs who are allowed to produce may choose not to produce because his productivity is too low. On the other hand, the inactive entrepreneurs cannot produce regardless of his productivity and his income is just $(1 + r)a$. Because of log utility and since the profit function is linear in wealth in either case, I can derive a closed form solution for the optimal savings policy function for producing entrepreneurs (a'_p) and inactive entrepreneurs (a'_I):

$$a'_p = \beta [\max \{z\pi - R, 0\} \lambda + 1 + r] a$$

$$a'_I = \beta [1 + r] a$$

E.4 Equilibrium and Aggregate TFP in the Steady State

An equilibrium in the steady state consists of prices r and w such that active entrepreneurs maximize [equation \(23\)](#) subject to [equation \(24\)](#) and inactive entrepreneurs maximize [equation \(25\)](#) subject to [equation \(26\)](#) taking the equilibrium prices as given, and that capital and labor markets clear. Let $g(a, z)$ be the joint distribution of entrepreneurs with a and

z . Denote the marginal distribution of wealth by $\phi(a)$. The market clearing conditions for capital and labor are given by

$$\int k(a, z)dG(a, z) = \int adG(a, z) \quad (27)$$

$$\int l(a, z)dG(a, z) = L \quad (28)$$

Using that $k = \lambda a$ for active entrepreneurs with $z \geq \underline{z}$, the fact that the joint distribution $g(a, z) = \phi(a)f(z)$, and the fact that a mass p of active entrepreneurs can produce, the market clearing condition for capital in [equation \(27\)](#) becomes

$$\lambda p(1 - F(\underline{z})) = 1 \quad (29)$$

which pins down \underline{z} as a function of λ and p . A lower value of λ (strong presence of financial frictions) means lower \underline{z} . Likewise, a lower p (higher exit rate) means lower \underline{z} . Finally, using the market clearing condition for labor and the optimal production decision of each entrepreneur, TFP is given by

$$Z = \left(\lambda p \int_{\underline{z}}^{\infty} z f(z) dz \right)^{\alpha} = \left(\frac{\int_{\underline{z}}^{\infty} z f(z) dz}{1 - F(\underline{z})} \right)^{\alpha} = (\mathbb{E}[z|z \geq \underline{z}])^{\alpha}$$

which is a truncated weighted average of productivities. We see immediately from the first equality how the exit probability, which I define as $(1 - p)$, is inversely related to TFP. If exit probability $(1 - p)$ is high, then aggregate TFP is low. If I only have information on firm-level data, p would be calibrated to the firm's exit rate, which I will denote by $1 - p_f$. Then TFP in this economy is given by

$$Z_f = (\mathbb{E}[z|z \geq \underline{z}(\lambda, p_f)])^{\alpha}$$

However, suppose p is calibrated to the *entrepreneur's* exit rate, denoted as $1 - p_e$, then all else constant, the aggregate TFP in this economy would be

$$Z_e = (\mathbb{E}[z|z \geq \underline{z}(\lambda, p_e)])^\alpha$$

And the relative TFP of the two economies would be

$$\frac{Z_e}{Z_f} = \left(\frac{\mathbb{E}[z|z \geq \underline{z}(\lambda, p_e)]}{\mathbb{E}[z|z \geq \underline{z}(\lambda, p_f)]} \right)^\alpha \quad (30)$$

In a world in which each entrepreneur owns one firm, firm's and entrepreneur's exit rate would be the same such that $p_e = p_f$ and there would be no relative gain in TFP. However, if entrepreneurs own multiple firms and their exit rate is lower, such that $p_e > p_f$, then there will be a TFP gain. To see why, note from [equation \(29\)](#) that $\underline{z}(\lambda, p_e) > \underline{z}(\lambda, p_f)$ for $p_e > p_f$. In other words, the threshold \underline{z} in which entrepreneurs operate is higher when $p_e > p_f$. Because truncated expectation $\mathbb{E}[z|z \geq \underline{z}]$ increases with \underline{z} , $Z_e > Z_f$. The intuition is that if entrepreneurs exit less, there will be more producing entrepreneurs, which in equilibrium will drive up the rental rate of capital. With a higher rental rate of capital, the cutoff productivity \underline{z} increases. As a result, higher survival rate p allows more productive entrepreneurs to survive, increasing aggregate TFP.

If an entrepreneur's exit rate is lower than the firm's exit rate — as in the case of the Thai data — then calibrating the data using the *firm's* exit rate will lead to a different aggregate implication than the calibration that uses the entrepreneur's exit rate. In this simple case in which productivity is i.i.d., calibrating to the firm's exit rate will lead to an overestimation of the effect of financial frictions.

However, unlike the simple model, the data shows that output is highly persistent, entrepreneurs have multiple firms, and an entrepreneur's exit is not random. Therefore, in a later section I explore a richer model in which I allow for persistence in productivity, entrepreneurs to have multiple firms, endogenous entry and exit.

References

- Akcigit, Ufuk, Harun Alp, and Michael Peters, “Lack of Selection and Limits to Delegation: Firm Dynamics in Developing Countries,” Working Paper 21905, National Bureau of Economic Research January 2016.
- , —, and —, “Lack of Selection and Limits to Delegation: Firm Dynamics in Developing Countries,” Working Paper 21905, National Bureau of Economic Research January 2016.
- Angeletos, George-Marios and Laurent-Emmanuel Calvet, “Incomplete-market dynamics in a neoclassical production economy,” *Journal of Mathematical Economics*, 2005, 41 (4), 407–438.
- Belenzon, Sharon, Aaron K Chatterji, and Brendan Daley, “Eponymous entrepreneurs,” Technical Report, Working paper 2014.
- , Andrea Pataconi, and Rebecca Zarutskie, “Married to the firm? A large-scale investigation of the social context of ownership,” *Strategic Management Journal*, 2015.
- Bertrand, Marianne, Simon Johnson, Krislert Samphantharak, and Antoinette Schoar, “Mixing Family With Business: A Study of Thai Business Groups and the Families Behind Them,” Working Paper 13738, National Bureau of Economic Research January 2008.
- Bloom, Nicholas, Benn Eifert, Aprajit Mahajan, David McKenzie, and John Roberts, “Does Management Matter? Evidence from India,” *The Quarterly Journal of Economics*, 2013, 128 (1), 1–51.
- Buera, Francisco J, “A dynamic model of entrepreneurship with borrowing constraints: theory and evidence,” *Annals of finance*, 2009, 5 (3-4), 443–464.
- Buera, Francisco J. and Benjamin Moll, “Aggregate Implications of a Credit Crunch: The Importance of Heterogeneity,” *American Economic Journal: Macroeconomics*, July 2015, 7 (3), 1–42.
- Buera, Francisco J and Yongseok Shin, “Financial frictions and the persistence of history: A quantitative exploration,” Technical Report, National Bureau of Economic Research 2010.

- Buera, Francisco J., Joseph P. Kaboski, and Yongseok Shin, “Finance and Development: A Tale of Two Sectors,” *American Economic Review*, August 2011, *101* (5), 1964–2002.
- Buera, Francisco J, Joseph P Kaboski, and Yongseok Shin, “Entrepreneurship and Financial Frictions: A Macro-Development Perspective,” Technical Report, National Bureau of Economic Research 2015.
- Caselli, Francesco and Nicola Gennaioli, “Dynastic management,” *Economic Inquiry*, 2013, *51* (1), 971–996.
- Clementi, Gian Luca and Berardino Palazzo, “Entry, exit, firm dynamics, and aggregate fluctuations,” 2013.
- Cooley, Thomas F and Vincenzo Quadrini, “Financial markets and firm dynamics,” *American economic review*, 2001, pp. 1286–1310.
- Covas, Francisco, “Uninsured idiosyncratic production risk with borrowing constraints,” *Journal of Economic Dynamics and Control*, 2006, *30* (11), 2167–2190.
- Decker, Ryan, John Haltiwanger, Ron Jarmin, and Javier Miranda, “The role of entrepreneurship in US job creation and economic dynamism,” *The Journal of Economic Perspectives*, 2014, *28* (3), 3–24.
- Evans, David S and Boyan Jovanovic, “An estimated model of entrepreneurial choice under liquidity constraints,” *The Journal of Political Economy*, 1989, pp. 808–827.
- Fenton, Lawrence, “The sum of log-normal probability distributions in scatter transmission systems,” *IRE Transactions on Communications Systems*, 1960, *8* (1), 57–67.
- Gopinath, Gita, Sebnem Kalemli-Ozcan, Loukas Karabarbounis, and Carolina Villegas-Sanchez, “Capital allocation and productivity in south europe,” 2015.
- hadj Bah, El and Lei Fang, “Entry Costs, Financial Frictions, and Cross-country Differences in Income and TFP,” *Macroeconomic Dynamics*, 2016, *20* (04), 884–908.

- Hopenhayn, Hugo A, “Entry, exit, and firm dynamics in long run equilibrium,” *Econometrica: Journal of the Econometric Society*, 1992, pp. 1127–1150.
- Hsieh, Chang-Tai and Peter J. Klenow, “Misallocation and Manufacturing TFP in China and India,” *The Quarterly Journal of Economics*, 2009, *124* (4), 1403–1448.
- and —, “The Life Cycle of Plants in India and Mexico,” *The Quarterly Journal of Economics*, 2014, *129* (3), 1035–1084.
- Jeong, Hyeok and Robert M Townsend, “Sources of TFP growth: occupational choice and financial deepening,” *Economic Theory*, 2007, *32* (1), 179–221.
- Jr, Robert E Lucas, “On the size distribution of business firms,” *The Bell Journal of Economics*, 1978, pp. 508–523.
- Kerr, Sari, William R Kerr, and Ramana Nanda, “House money and entrepreneurship,” Technical Report, National Bureau of Economic Research 2015.
- Li, Huiyu et al., “Leverage and Productivity,” *Unpublished manuscript: Stanford University*, 2015.
- Matsuyama, Kiminori, “Aggregate implications of credit market imperfections,” in “NBER Macroeconomics Annual 2007, Volume 22,” University of Chicago Press, 2008, pp. 1–60.
- Midrigan, Virgiliu and Daniel Yi Xu, “Finance and Misallocation: Evidence from Plant-Level Data,” *American Economic Review*, February 2014, *104* (2), 422–58.
- Moll, Benjamin, “Productivity Losses from Financial Frictions: Can Self-Financing Undo Capital Misallocation?,” *American Economic Review*, October 2014, *104* (10), 3186–3221.